Theoretical Methods Problem Sheet 2: Spring Term

Series, Power Series Expansions and Thermodynamics

Power Series

The Maclaurin series expansion of a function f(x) was introduced in lecture 3.

$$f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots + c_n x^n + \ldots$$

Where the coefficients can be computed from the derivatives of the target function;

$$c_0 = f(0)$$

$$c_1 = \left.\frac{df}{dx}\right|_{x=0}$$

$$c_2 = \left.\frac{1}{2!}\frac{d^2 f}{dx^2}\right|_{x=0}$$

$$c_3 = \left.\frac{1}{3!}\frac{d^2 f}{dx^2}\right|_{x=0}$$

$$\ldots$$

$$c_n = \left.\frac{1}{n!}\frac{d^n f}{dx^n}\right|_{x=0}$$

Reminder: 6! Simply means 6x5x4x3x2x1.

(i) Use the Maclaurin series to obtain the power series expansion of cos(x) and sin(x) up to n=7. Why does the cos(x) series contain only even powers of x and the sin(x) series contain only odd powers ?

(ii) How many terms of the series are required to obtain sin(π/3) and cos(π/3) to 3 decimal places.

In the vibrations of solids (phonons) and the electronic structure of semiconductors you will often come across periodic functions which can be written in terms of sin(x) and cos(x) but are more usually written in terms of the exponential function by using the famous Euler identity;

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Reminder: i is the complex number and is defined so that i^2=-1

(iii) Make an Euler expansion of e^x to 6 terms and use it with your expansion of cos and sin from question (i) to prove the Euler identity.